1). (Griffiths 3.22) Consider a three-dimensional vector space spanned by an orthonormal basis $|e_1\rangle$, $|e_2\rangle$, $|e_3\rangle$. Two ket vectors are given by

$|\alpha\rangle = i|e_1\rangle - 2|e_2\rangle - i|e_3\rangle$,
$|\beta\rangle = i|e_1\rangle + 2|e_3\rangle$.

(a) Construct $\langle \alpha |$ and $\langle \beta |$ in terms of the dual basis $\langle e_1 |$, $\langle e_2 |$, $\langle e_3 |$.

(b) Find $\langle \alpha | \beta \rangle$ and $\langle \beta | \alpha \rangle$, and confirm that $\langle \beta | \alpha \rangle = \langle \alpha | \beta \rangle^*$.

(c) Find all nine matrix elements $\Omega_{mn}$, $m, n = 1, 2, 3$ of the operator $\Omega = |\alpha\rangle \langle \beta |$ in this basis and construct that matrix $\Omega$. Is it Hermitian?

2). Find the eigenvalues and the normalized eigenvectors for a matrix operator $\Omega$,

$$
\Omega = \begin{pmatrix}
2 & 1 & 1 \\
1 & 0 & -1 \\
1 & -1 & 2
\end{pmatrix}.
$$

3). Apply the normalized eigenvectors in Problem 2; construct a unitary matrix $U$. Then diagonalize $\Omega$ with the unitary matrix $U$.

4). Determine the value of the constant $B$ for

$$
t_b(x) = \begin{cases}
0, & \text{for } x^2 > b^2, \\
B(b - |x|) & \text{for } x^2 \leq b^2, \quad b > 0,
\end{cases}
$$

such that

$$\lim_{b\to0} t_b(x) = \delta(x).$$